Lab 5: Analysis of an ODE system

Due date: Thursday Feb 19, 11:59pm
Analyze this particular set of ODE:

$$\frac{dL}{dt} = \alpha L(1-\theta_1) - \gamma L \tag{1}$$

$$\frac{dE}{dt} = \beta E(1-\theta_2) - \delta E \tag{2}$$

with constraints so that

$$\theta_1, \theta_2 \begin{cases} \frac{L}{K_1}, \frac{E}{K_2} & \frac{L}{K_1} \geq \frac{E}{K_2} \\ \frac{L+E}{K_1+K_2} & \text{otherwise} \end{cases} \tag{3}$$

The parameters α and β are reproduction rates and the parameters γ and δ are death parameters. L and E are the numbers of the species Rana lessonae and Rana esculenta. These frogs interbreed. The regions K_1 and K_2 are locations in a pond where the different species lay their eggs. You can see that these regions are conditioning the ODE and you will need to take care of that (for example using global variables).
Give an analysis of this system, use the tutorial on the Lotka-Volterra as guide[1], but remember that we use our own ODE system and you will need to invent a way to show the arrow graph, think of a *for* loop]. We do not expect identical analysis but you should give an idea about the behavior of the formulae, for example a plot of the relationship to time and the relationship of the two variables L and E are needed, The example plot shows a grid with "arrows" that shows that for the particular configuration used the L will go extinct. Mathematicians tell me that there is actually a stable configuration where L and E do not go extinct, although I had difficult to show that. You may want to explore the situations:

A $\frac{\alpha}{\gamma} < \frac{\beta}{\delta}$

B $\frac{\alpha}{\gamma} > \frac{\beta}{\delta}$

The plot in figure 1 was generated using $\alpha = 0.09, \beta = 0.06, \gamma = 0.05, \delta = 0.04, K_1 = 200, K_2 = 200$, and initial condition $L = 100, E = 100$ I expect that you show plots that describe the behavior of the ODE and show your code.

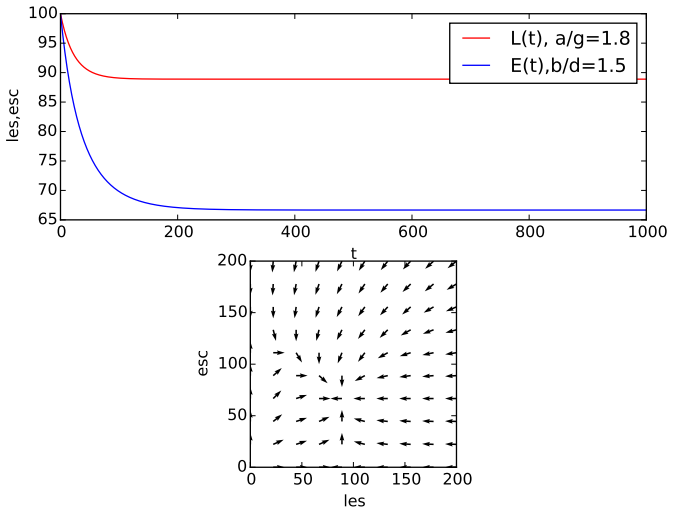

Figure 1: 'Lousy' example plot, better would be to use `gridspec` (http://matplotlib.org/users/gridspec.html), and also 'plot' the parameters used.

[1] Look at: http://wiki.scipy.org/Cookbook/LoktaVolterraTutorial