Lab 4: Random Processes

Using Monte Carlo to integrate a function

In this lab we intend to integrate the following function using Monte Carlo

$$f(x,y) = \exp\left(\frac{-x^2}{2}\right)\exp\left(\frac{-y^2}{2}\right) + \exp\left(\frac{-(x-3)^2}{10}\right)\exp\left(\frac{-(y-3)^2}{10}\right)$$

over the domain of $[-5, 10]$ for x and $[-10, 10]$ for y. Therefore we will need to solve

$$\int_{x=-5}^{x=10}\int_{y=-10}^{y=10} f(x,y)dxdy$$

Using MC we simply draw a random coordinate (x, y, z) in a box that is bigger than the function. We can then calculate the ratio between the volume of the box and the volume under the function and multiply this ratio with the volume of the box (similary to the 1-D evaluation in the lecture material). Mathematica calculates the volume under the function as 37.6389.

1. Plot the function
2. Develop a Monte Carlo program that uses the Monte Carlo rejection method to calculate the volume.
3. Test your program with different numbers of samples, for example $n = [100, 500, 1000, 5000, 10000, ...]$. Pick n that you feel are sensible to show your work.
4. For each n evaluate five runs and calculate and report the mean and standard deviation, in a table that has three columns n, Volume mean, Volume standard deviation.
5. Evaluate the convergence to the correct results by plotting on a log log scale the absolute difference between the mathematica result and your mean estimates (this is the y-axis and n (the x-axis). Plot also the theoretical convergence rate $1/\sqrt{n}$ into the same plot using a different color.
6. Discuss your plots and observation about the error.